

****FULL TITLE****

*ASP Conference Series, Vol. **VOLUME**, **YEAR OF PUBLICATION***

****NAMES OF EDITORS****

Non Kolmogorov-like Turbulence in the Local Interstellar Medium

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Abstract. We develop a self-consistent model of turbulence in a local interstellar medium (ISM). The model describes a partially ionized magnetofluid ISM in which a neutral hydrogen fluid interacts with a plasma dominantly through a charge exchange. The ISM turbulent correlation scales in our model are much bigger than the shock characteristic length-scales. Unlike small length-scale linear collisional dissipation in the fluid, the charge exchange processes can be effective unpredictably on a variety of ISM length-scales depending upon the neutral and plasma densities, the charge exchange cross section and the characteristic length scales. We find, from scaling arguments that the charge exchange interactions modify spectral transfer associated with large-scale energy containing eddies. Consequently, the ISM turbulent cascade are steeper than those predicted by Kolmogorov's phenomenology.

1. Introduction and Model Equations

Small scale turbulence in the local interstellar medium (ISM) is a largely unexplored field given the complexity associated with the ISM turbulent processes (Shaikh *et. al.* 2006). The local ISM comprises of partially ionized, magnetized plasma protons and almost equal number of neutral particles. The plasma and the neutral particles in ISM interact mutually through charge exchange. The physics of these small-scale turbulent motions is far more complex than ever thought. Not only that it holds the key to our crucial understanding of the global heliospheric interactions such as nature and characteristic of heliospheric shocks, heating etc (Zank 1999; Pauls *et. al.* 1995), it is also increasingly believed to be pivotal to many puzzles of astrophysics including origin and transport of cosmic rays, Fermi acceleration, gamma-ray bursts, ISM density spectra etc. Yet, there exists *no* self-consistent simulation model that unravels multi-component and multiple-scale ISM turbulent phenomena. A prime goal of this paper is therefore to develop a self-consistent plasma-neutral ISM turbulence model based on analytic methods and numerical simulations.

The underlying model is based on the following assumptions. Fluctuations in the plasma and the neutral fluids are sufficiently isotropic, homogeneous, thermally equilibrated and turbulent. No mean magnetic field and velocity flows are present at the outset. There however may generate local mean flows due to self-consistently excited nonlinear instabilities. The characteristic turbulent correlation length-scales are typically smaller than charge-exchange mean free path lengths in the ISM flows. Nevertheless, they are large enough to treat any localized shocks as discontinuities. In other words, the characteristic shock length-scales are relatively small compared to the ISM turbulent fluctuation

length-scales, and finally boundary conditions are periodic. It should be further noted that the neutrals coming from the solar wind are not considered because they tend to anisotropize the distribution functions substantially. Our model thus simulates a localized ISM. The fluid model describing nonlinear turbulent processes in the interstellar medium, in the presence of charge exchanges forces, can then be cast into plasma density (ρ_p), velocity (\mathbf{U}_p), magnetic field (\mathbf{B}), pressure (P_p), as follows.

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p \mathbf{U}_p) = 0, \quad (1)$$

$$\rho_p \left(\frac{\partial}{\partial t} + \mathbf{U}_p \cdot \nabla \right) \mathbf{U}_p = -\nabla P_p + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \mathbf{Q}_M(\mathbf{U}_p, \mathbf{V}_n), \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U}_p \times \mathbf{B}), \quad (3)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \left(\frac{1}{2} \rho_p U_p^2 \mathbf{U}_p + \frac{\gamma}{\gamma - 1} \frac{P_p}{\rho_p} \rho_p \mathbf{U}_p + \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} \right) = Q_E(\mathbf{U}_p, \mathbf{V}_n). \quad (4)$$

where $e = 1/2 \rho_p U_p^2 + P_p/(\gamma - 1) + B^2/8\pi$. The above set of plasma equations is coupled self-consistently to the ISM neutral density (ρ_n), velocity (\mathbf{V}_n) and pressure (P_n) through a set of hydrodynamic fluid equations as below.

$$\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n \mathbf{V}_n) = 0, \quad (5)$$

$$\rho_n \left(\frac{\partial}{\partial t} + \mathbf{V}_n \cdot \nabla \right) \mathbf{V}_n = -\nabla P_n + \mathbf{Q}_M(\mathbf{V}_n, \mathbf{U}_p), \quad (6)$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho_n V_n^2 + \frac{P_n}{\gamma - 1} \right) + \nabla \cdot \left(\frac{1}{2} \rho_n V_n^2 \mathbf{V}_n + \frac{\gamma}{\gamma - 1} \frac{P_n}{\rho_n} \rho_n \mathbf{V}_n \right) = Q_E(\mathbf{V}_n, \mathbf{U}_p). \quad (7)$$

Equations (1) to (7) form an entirely self-consistent description of the coupled ISM plasma-neutral turbulent fluid. The charge-exchange momentum sources in the plasma and the neutral fluids are described respectively by terms $\mathbf{Q}_M(\mathbf{U}_p, \mathbf{V}_n)$ and $\mathbf{Q}_M(\mathbf{V}_n, \mathbf{U}_p)$. Similarly, charge exchange energy sources in the plasma and the neutral fluids are given by $Q_E(\mathbf{U}_p, \mathbf{V}_n)$ and $Q_E(\mathbf{V}_n, \mathbf{U}_p)$ respectively. In the absence of charge exchange interactions, the plasma and the neutral fluid are de-coupled trivially and behave as ideal fluids. While the charge-exchange interactions modify instantaneous momentum and the energy of plasma and the neutral fluids, they tend to conserve density in both the fluids. Nonetheless, the volume integrated energy ($\int Q_E(\mathbf{U}_p, \mathbf{V}_n) d\mathbf{v} = -\int Q_E(\mathbf{V}_n, \mathbf{U}_p) d\mathbf{v}$) and the density ($\int \rho_p d\mathbf{v} = \text{const}$, $\int \rho_n d\mathbf{v} = \text{const}$) of the entire coupled system will remain conserved in the absence of external sources/sinks.

The ISM turbulence model is normalized using the typical ISM scale-length (ℓ_0), density (ρ_0) and velocity (v_0). The normalized plasma density, velocity, energy and the magnetic field are respectively; $\bar{\rho}_p = \rho_p/\rho_0$, $\bar{\mathbf{U}}_p = \mathbf{U}_p/v_0$, $\bar{P}_p = P_p/\rho_0 v_0^2$, $\bar{\mathbf{B}} = \mathbf{B}/v_0 \sqrt{\rho_0}$. The corresponding neutral fluid quantities are $\bar{\rho}_n = \rho_n/\rho_0$, $\bar{\mathbf{U}}_n = \mathbf{U}_n/v_0$, $\bar{P}_n = P_n/\rho_0 v_0^2$. The momentum and the energy charge-exchange forces, in the normalized form, are respectively $\bar{\mathbf{Q}}_m = \mathbf{Q}_m \ell_0/\rho_0 v_0^2$, $\bar{Q}_e =$

$Q_e \ell_0 / \rho_0 v_0^3$. The non-dimensional temporal and spatial length-scales are $\bar{t} = t v_0 / \ell_0$, $\bar{\mathbf{x}} = \mathbf{x} / \ell_0$. The charge-exchange cross-section parameter (σ), not appeared directly in the above set of equations, is normalized as $\bar{\sigma} = n_0 \ell_0 \sigma$. We define an intrinsic mode in ISM turbulence, the charge exchange mode $k_{ce} \sim (n_0 \ell_0)^{1/2}$. The latter is different from the characteristic turbulent mode k , and is excited naturally when plasma and neutral fluids are coupled in the ISM by charge exchange forces [see Shaikh *et. al.* (2006) for more detail].

2. Nonlinear Simulation

Two-dimensional (2D) nonlinear spectral fluid code is developed to numerically integrate Eqs. (1) to (7). The 2D simulations are not only computationally simpler and less expensive (compared with the full 3D), they offer significantly higher resolutions even on moderate size small cluster machines like Beowulf. The spatial discretization in our code uses discrete Fourier representation of turbulent fluctuations based on a spectral method, $\tilde{f}(\mathbf{x}, \mathbf{t}) = \sum_{\mathbf{k}} f(\mathbf{k}, t) e^{-i\mathbf{k} \cdot \mathbf{x}}$, where $\mathbf{k} = k_x \hat{e}_x + k_y \hat{e}_y$ is a two-dimensional wave vector. The nonlinear deconvolution of Fourier modes is performed by computing nonlinear triad interaction $\tilde{f}(\mathbf{x}, \mathbf{t}) \tilde{g}(\mathbf{x}, \mathbf{t}) = \sum_{\mathbf{k}=\mathbf{k}'-\mathbf{k}''} f(\mathbf{k}', t) g(\mathbf{k}'', t) \delta(\mathbf{k}' - \mathbf{k}'')$ which survive for only those mode coupling interactions which satisfy Fourier triad constraint $\mathbf{k} = \mathbf{k}' - \mathbf{k}''$. These nonlinear interactions in Fourier space conserve rugged invariants of system of Eqs. (1) to (7). The temporal integration is performed by Runge Kutta 4 method. The fluctuations in plasma and neutral are initialized isotropically (no mean fields are assumed) with random phases and with identical amplitudes in the Fourier space. This algorithm ensures conservation of total energy and mean fluid density per unit time in the absence of charge exchange and external random forcing. Additionally, it satisfies the condition of incompressibility associated with the magnetic field, i.e. $\int |\nabla \cdot \mathbf{B}|^2 d\mathbf{v} \approx 10^{-15}$ at each time step. We make use of an artificial scalar potential to achieve $\nabla \cdot \mathbf{B} = 0$ as described in Kulikovskii *et al.* (2001). Numerical resolution is 512^2 for the mode structures and 1024^2 for the spectrum calculations. Our code is massively parallelized using Message Passing Interface (MPI) libraries to facilitate higher resolution. While the ISM turbulence code is evolved with time steps resolved self-consistently by the coupled fluid motions, the nonlinear interaction time scales associated with the plasma $1/\mathbf{k} \cdot \mathbf{U}_p(\mathbf{k})$ and the neutral $1/\mathbf{k} \cdot \mathbf{V}_n(\mathbf{k})$ fluids can obviously be disparate. Accordingly, turbulent transport of energy in the plasma and the neutral ISM fluids takes place distinctively on separate time scales. Small scale turbulent fluctuations in ISM evolve under the action of nonlinear interactions as well as charge exchange sources. Energy cascades amongst turbulent eddies of various scale sizes and between the plasma and the neutral fluids. Because of the discrepant nonlinear time scales associated with the plasma and neutral fluids, mode structures can be different in the two fluids when they are evolved together and in isolation (i.e. decoupled). When the plasma and neutral components are decoupled, the plasma fluid evolves in accordance with the ideal MHD where current sheets are typically formed in the magnetic field structures. In the presence of charge exchange, the two fluids evolve in a self-consistent manner and modify the dynamical properties of both the fluids. One of the most noticeable features to emerge from the coupled evolution is that the spectral cascade

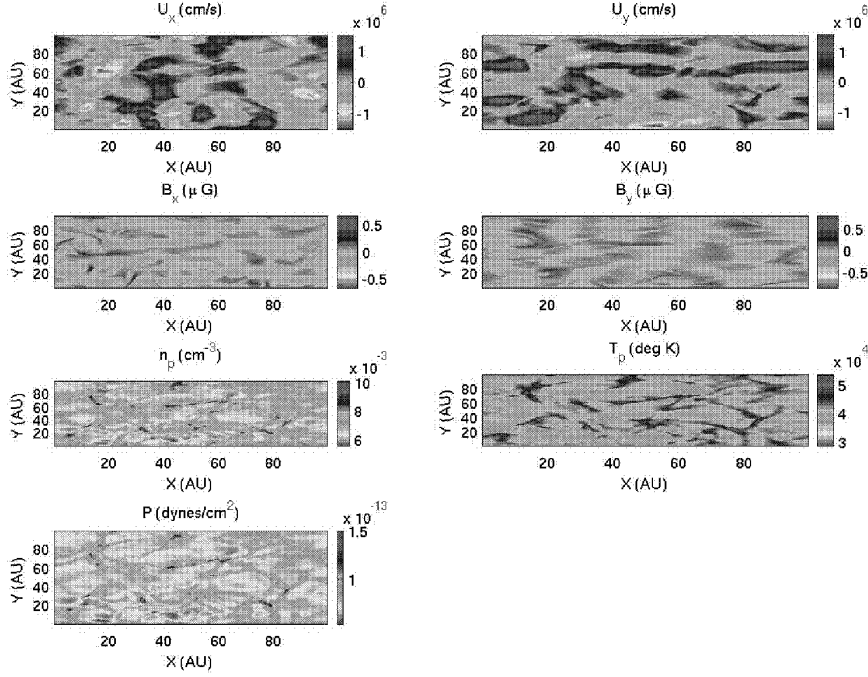


Figure 1. Self-consistent simulations of turbulence in the partially ionized ISM. Figure shows evolution of the plasma density, velocity, magnetic field, temperature and pressure. Numerical resolution is 512^2 .

rates are enhanced substantially in contrast to that of the decoupled case. The rapid spectral transfer of energy amongst various Fourier modes in the coupled plasma neutral gas leads to smearing off the current sheets in the plasma on a shorter time scales. The latter evolves on a relatively long time scales and the sheets are formed eventually in the magnetic field. The turbulent equipartition is set up in the plasma and the neutral fluid modes. This is shown in Fig 1 where all the quantities associated with the plasma evolution in our simulations are depicted. Seemingly, the neutral fluid, under the action of charge exchange sources, tends to enhance the cascade rates by isotropizing the ISM turbulence on rapid time scales. One of the plausible reasons of the fast-cascade is that the energy transfer rates are enhanced effectively in the coupled ISM turbulence system, and that they now require a smaller time to mix the plasma and the neutral fluids. In any event, the small-scale sheet-like structures in magnetic field compresses (or pinches) the plasma density. Hence the density fluctuations develop identically the (thinner than) sheet-like structures that co-exist with small-scale turbulent fluctuations in its spectrum as shown in Fig 1. The neutral fluid, on the other hand, evolves isotropically as stated above by forming relatively large-scale structures as shown in Fig 2.

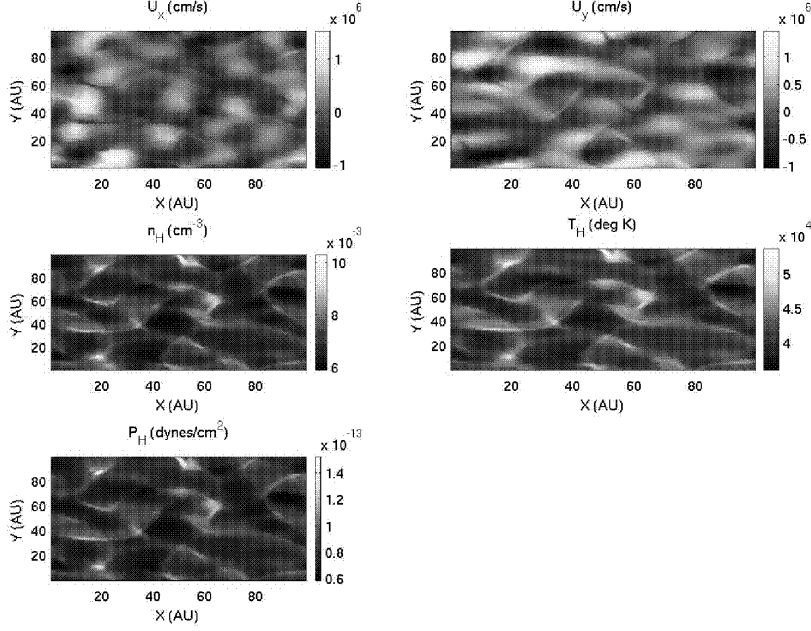


Figure 2. Evolution of neutral fluid in the coupled plasma neutral ISM turbulence. Shown are the neutral density, velocity, temperature and pressure fluctuations.

3. Energy Cascade Rates in the Coupled ISM Plasma Neutral Gas

As mentioned above, the plasma neutral fluid coupling in the ISM fluctuations introduces charge exchange modes, k_{ce} , that are distinctively different from the characteristic turbulent mode k . Typically, $k_{ce}/k < 1$ in the local ISM. Accordingly, the energy cascade time scales are modified as discussed below. The nonlinear interaction time-scale in ordinary turbulence is given by

$$\tau_{nl} \sim \frac{\ell_0}{v_\ell} \sim (kv_k)^{-1},$$

where v_k or v_ℓ is the velocity of turbulent eddies. In the presence of charge exchange interaction, the ordinary nonlinear interaction time-scales of fluid turbulence is modified by a factor k_{ce}/k such that the new nonlinear interaction time-scale of ISM turbulence is now

$$\tau_{NL} \sim \frac{k_{ce}}{k} \frac{1}{kv_k}.$$

On using the fact that k_{ce} is typically smaller than k , i.e. $k_{ce}/k < 1$ in the ISM, the new nonlinear time is k_{ce}/k times smaller than the old nonlinear time i.e. $\tau_{NL} \sim (k_{ce}/k)\tau_{nl}$. This reduced nonlinear interaction time in ISM turbulence is likely to enhance turbulent energy cascade rates that are determined typically by E_k/τ_{NL} , where E_k is energy per unit mode. It is because of this reduced

interaction time that a rapid spectral transfer of turbulent modes tends to smear off the current sheets in the magnetic field fluctuations.

It is interesting to note that the enhanced cascade rates of ISM turbulent modes tend to steepen the inertial range turbulent spectra in both plasma and neutral fluids. By extending above phenomenological analysis, one can deduce exact (analytic) spectral indices of the inertial range decaying turbulent spectra, as follows. The new nonlinear interaction time-scale of ISM turbulence can be rearranged as $\tau_{NL} \sim k_{ce}v_k/(kv_k)(1/kv_k) \sim (\tau_{nl}^2/\tau_{ce})$, where $\tau_{ce} \sim (k_{ce}v_k)^{-1}$ represents charge exchange time scale. The energy dissipation rate associated with the coupled ISM plasma-neutral system can be determined from $\varepsilon \sim E_k/\tau_{NL}$ relation, which leads to $\varepsilon \sim v_k^2/(k_{ce}/k^2v_k) \sim k^2v_k^3/k_{ce}$. According to the Kolmogorov theory, the spectral cascades are local in k -space and the inertial range energy spectrum depends upon the energy dissipation rates and the characteristic turbulent modes, such that $E_k \sim \varepsilon^\gamma k^\beta$. Upon substitution of above quantities and equating the power of identical bases, one obtains

$$E_k \sim \varepsilon^{2/3} k^{-7/3}$$

plasma spectrum. Similar arguments in the context of neutral fluids, when coupled with the plasma fluid in ISM, lead to the energy dissipation rates $\varepsilon \sim k^2v_k^2/(k_{ce}/k^2v_k)$. This further yields the forward cascade (neutral) energy spectrum

$$E_k \sim \varepsilon^{2/3} k^{-11/3}.$$

4. Conclusion

In conclusion, one of the most important points to emerge from our studies is that the charge exchange modes modify the ISM turbulence cascades dramatically by expediting nonlinear interaction time-scales. Consequently, the energy cascade rates are enhanced by isotropizing the ISM turbulence on rapid time scales. This tends to modify the characteristics of ISM turbulence which can be significantly different from the Kolmogorov phenomenology of fully developed turbulence. As shown above, the charge exchange interactions lead to a steeper power spectrum. Although turbulence in 2D and 3D possesses distinct spectral features characterized essentially by the number of the inviscid quadratic invariants, our 2D simulations provide a hint that the ISM turbulent spectra in 3D case will exhibit a non Kolmogorov-like characteristic. It is to be noted that the present model does not consider driving mechanism, hence turbulence is freely decaying. Driven turbulence, such as due to large scale forcing, supernova explosion and CME ejection may force turbulence at larger scales. This can modify the cascade dynamics in a manner usually described by dual cascade process.

References

- Zank, G. P. 1999, Sp. Sci. Rev., 89, 413
 Pauls, H. L., Zank, G. P., & Williams, L. L. 1995 J. Geophys. Res. A11, 21595
 Shaikh, D. Pogorelov, N. & Zank, G. P., 2006, AIP Conf. Procs, **858**, 314.
 Kulikovskii, A. G., Pogorelov, N. V., and Semenov, A. Y., *Mathematical Aspects of Numerical Solution of Hyperbolic Systems*, Chapman & Hall/CRC, 118, Pg. 50.